Volume 21 (2002), Number 4 pp. 1–7

Vortex Tracking and Visualisation in a Flow Past a Tapered Cylinder

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Abstract

In this paper we explore a novel combined application of two of our existing visualisation techniques to the tracking of 3D vortex tubes in an unsteady flow. The applied techniques are the winding-angle vortex extraction technique based on streamline geometry, and the attribute-based feature tracking technique. We have applied these to the well-known case of an unsteady 3D flow past a tapered cylinder.
First, 2D vortices are detected in a number of horizontal slices for each time step, by means of the winding-angle vortex extraction method. For each 2D vortex a number of attributes are calculated and stored. These vortices are visualised by a special type of ellipse icons, showing the position, shape, and rotational direction and speed in each slice.
Next, for each time step, 3D vortex tubes are constructed from the 2D vortices by applying the feature tracking procedure in a spatial dimension to connect the corresponding vortices in adjacent slices. The result is a graph attribute set with the 2D vortex attributes in the nodes and the spatial correspondences as edges.
Finally, the 3D vortex tubes are tracked in time using the same tracking procedure, for finding the corresponding tubes in successive time steps. The result is a description of the evolution of the 3D vortices. An interactive, time-dependent visualisation is generated using the temporal correspondences of each vortex tube. This analysis reveals a number of interesting patterns.

Categories and Subject Descriptors (according to ACM CCS): I.3.8 [Computer Graphics]: Applications

1. Introduction

Unsteady 3D flow simulations usually produce very large time-dependent data sets, which are difficult to analyse and visualise. A solution to this problem is the use of feature-based visualisation, in which significant phenomena (such as vortices) are detected in the data, and quantitatively described by computing attribute sets, which can be used for visualisation and further investigation. This approach concentrates on the high-level phenomena rather than on the low-level data, and the amount of data to be handled is reduced by a factor of 1000 or more[12].

Vortices are important features in fluid dynamics research. Therefore, the dynamics of vortices is investigated: their formation, interaction, breakdown, and dissipation, and the measurement of their strength and motion paths. A classical case is the shedding of vortices in the wake of a cylinder, which generates the so-called Von Karman vortices, a periodic series of vortices with alternating rotational directions, breaking away from the back of the cylinder surface in a regular pattern, depending on the radius of the cylinder. If the cylinder has a constant radius, the motion pattern of the vortices is periodic and essentially planar, and can be studied by a 2D simulation. If the cylinder's radius is variable, the vortical patterns are no longer periodic and planar, but they become unsteady and three-dimensional. We have studied the 3D vortex dynamics of the flow past a tapered cylinder, an unsteady flow data set publicly available from NASA Ames Research Center[2].

For the extraction of vortices, many techniques are available based on many different definitions of the vortex concept[9]. A very common technique is interactive generation of time-dependent particle paths, which can reveal vortices by showing swirling patterns. However, this technique can only be used for visual feature detection. Some of the algorithmic detection techniques try to formalize the concept of a swirling pattern, to automatically extract the vortices.

Our 2D vortex detection technique[11] is based on the definitions of Robinson[8] and Portela[5], and considers a vortex as a regional feature. We have used our detection technique based on streamline geometry, which has the advantage of finding slow vortices, and which allows a quantification of the vortices detected.

The vortex detection method works only in 2D, and we want to apply it to the 3D tapered cylinder flow. For that purpose, we use our feature tracking techniques[6,7] on the vortices extracted by the winding-angle method. We have applied the feature tracking twice: first to find the correspondences between vortices extracted from adjacent 2D horizontal slices, to construct 3D vortex tubes. Next, we have used it to track the 3D vortex tubes in time by finding the correspondences between the vortex tubes in successive frames.

The main purpose of this study is to show the effectiveness of combining 2D vortex detection with spatial and temporal tracking techniques, to obtain quantitative 3D time-dependent feature descriptions.

This paper presents some details and results of the combined application of the two methods to vortex tracking in the tapered cylinder data set, and some extensions of the feature tracking technique used for this purpose.

Section 2 describes the flow case. Sections 3 and 4 describe the techniques for vortex extraction and feature tracking, after which Section 5 describes the visualisation methods used. We discuss the procedure and the results of vortex tracking in Section 6. Finally, Section 7 contains a discussion of the results and future work.

2. Flow Past a Tapered Cylinder

The application is a CFD simulation of an unsteady 3D flow past a tapered cylinder, as described by Jespersen and Levit[2]. The geometry of the tapered cylinder (see Figure 1) results in interesting 3D flow phenomena in the wake of the cylinder. Behind the cylinder unsteady vortex shedding becomes apparent. The vortices alternatingly leave the left and right sides of the cylinder, and also alternatingly have clockwise and counterclockwise rotational directions. The vortex shedding frequency f depends, among other things, on the cylinder's diameter d which is a linear function of the z-coordinate.

We have obtained time-dependent data from NASA Ames Research Center[†]) of a flow past a tapered cylinder. The cylinder has a taper ratio of 0.01. The data is defined on a structured, cylindrical grid with $64 \times 64 \times 32$ nodes, each of which contains density, x-, y-, z-momentum, and stagnation. The complete data set has thousands of time-steps from which we used 400, from $t = 12000$ to $t = 16000$ with an increment of 10. Each time-step is stored in a file that is 2.6 Mb in size, so the total data set occupies over 1 Gb of disk space.

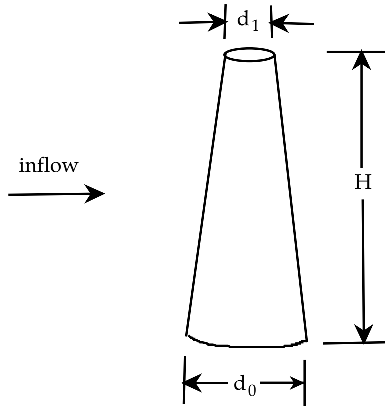

Figure 1: *A sketch of the geometry of the tapered cylinder (taper is greatly exaggerated), taper ratio* $= (d_0 - d_1)/H$.

Figure 2 shows a visualisation of the flow in the wake of the cylinder, using both standard visualisation techniques and feature-based techniques. Streamlines are shown to indicate the flow direction. It is clearly visible that the flow spirals up behind the cylinder. The icons show the vortex tubes we have detected, with the techniques described in this article.

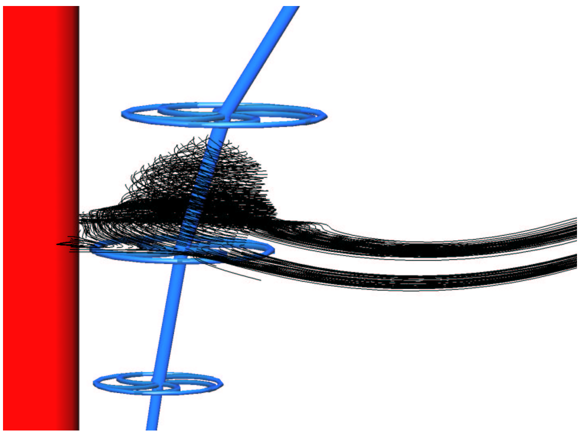

Figure 2: *The flow past a tapered cylinder. Streamlines are shown, together with an iconic representation of the vortex tubes detected.*

† http://www.nasa.gov/Research/Datasets/datasets.html

3. Winding-Angle Vortex Extraction

The vortical patterns in the flow past the tapered cylinder can be extracted in horizontal slices using the *winding-angle* technique[11].

It should be mentioned that one condition must be met for this method to be applicable. The projection of the flow streamlines onto the horizontal slices should result in a stable circulating pattern. In this application, the vortex cores are all roughly vertical, therefore the condition holds.

If the vortices are strongly curved, or oriented in any direction, multiple perpendicular slice planes can be used. In that way, all vortices can be detected by combining the results from the different slice planes. Ideally, slice planes should be used that are perpendicular to the vortex core. However, the location and orientation of the core will usually not be known in advance.

Other techniques are known for finding vortex core lines independent of direction and for finding curved vortex core lines (Roth and Peikert[9] and Kenwright and Haimes[3]), but none of these techniques results in quantitative descriptions that can be used for tracking the vortices in time. Because the winding-angle method does result in quantitative descriptions and because the condition mentioned above is known to be true for the tapered cylinder case, we have decided to use the winding-angle method.

The winding-angle technique works as follows: streamlines are calculated in a 2D plane which should be taken roughly perpendicular to the (expected) vortex core. Streamlines are seeded at points on a regular grid, such that the vortical regions are sufficiently covered by streamlines. Looping streamlines are selected from all streamlines by testing the winding-angle. The winding-angle is the sum of the angles between all pairs of consecutive line segments of a streamline (see Figure 3). The selection criteria for the streamlines are: the winding-angle $|\alpha_{w,i}| \geq 2\pi$, and the distance between the starting point and the final point is relatively small compared to the average radius. The latter criterion is used to ensure locality.

Note that in this technique, vortices are extracted from the streamlines calculated from an instantaneous velocity field for each frame. An alternative would be the use of time-dependent particle pathlines generated over a time interval, corresponding to a sequence of frames. These curves would represent the actual motions of fluid particles over the time interval. But these motion paths would reflect not only the swirling motion around the vortex core, but also the motion of the core itself, resulting in cycloidal curves which may not clearly show the vortical patterns. Another alternative would be the use of streaklines instead of streamlines. Streaklines are created by connecting all particles that have passed through a given point, and can be compared with injecting smoke or dye in the flow. Because both streaklines and pathlines are generated over a number of time steps, they

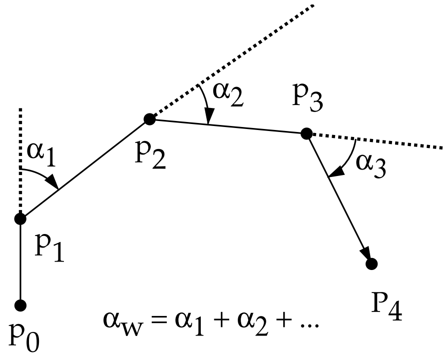

Figure 3: *The winding-angle $\alpha_{w,i}$ is the sum of the angles between the line segments of a streamline, [Sadarjoen & Post, 2000].*

will also show the motion of the vortex core itself. This will not be the case for streamlines. Therefore, we have used the instantaneous fields in which the streamlines only show the swirling patterns of the vortical flow.

After the selection of streamlines, the selected streamlines are clustered in order to group the streamlines belonging to the same vortex. The clustering is based on the centre point or the geometric mean of the streamline positions P_j. Once clustered, the streamlines in each cluster are used to quantify the vortex. The 2D shape of the vortices is approximated by ellipses. In addition, a number of specific vortex attributes can be calculated: the vortex rotational direction, and vortex angular velocity. The attributes are calculated as shown in Table 1, where $|S_i|$ is the number of points in streamline S_i, $C_k = \{S_{k,1}, S_{k,2}, \cdots\}$ is a cluster of streamlines, $S_{k,l}$ is streamline l in cluster k, $|C_k|$ is the number of streamlines in that cluster, and $\Psi(C_k)$ are all the points on all the streamlines in cluster k.

streamline centre:	$\vec{S_i} = \frac{1}{\lvert S_i\rvert}\sum_{j=1}^{\lvert S_i\rvert} \vec{P_{i,j}}$
cluster centre:	$\vec{C_k} = \frac{1}{\lvert C_k\rvert}\sum_{l=1}^{\lvert C_k\rvert}\left(\vec{S}_{k,l}\right)$
cluster covariance:	$M_k = \mathrm{cov}\left(\Psi(C_k)\right)$
ellipse axis lengths:	$\lambda_k = \mathrm{eig}\left(M_k\right)$
ellipse axis directions:	$\mathbf{d}_k = \mathrm{eigvec}\left(M_k\right)$
vortex rotational direction:	$d_k = \mathrm{sign}\left(\alpha_{w,k}\right)$
vortex angular velocity:	$\omega_k = \frac{1}{\lvert C_k\rvert \Delta t}\sum_{l=1}^{\lvert C_k\rvert} \alpha_{w,l}$

Table 1: *Vortex attributes calculated with the winding-angle method.*

Figure 4 shows two vortices extracted with the winding-angle method in a single horizontal slice. Two vortices with opposite rotational directions appear in the wake of the tapered cylinder.

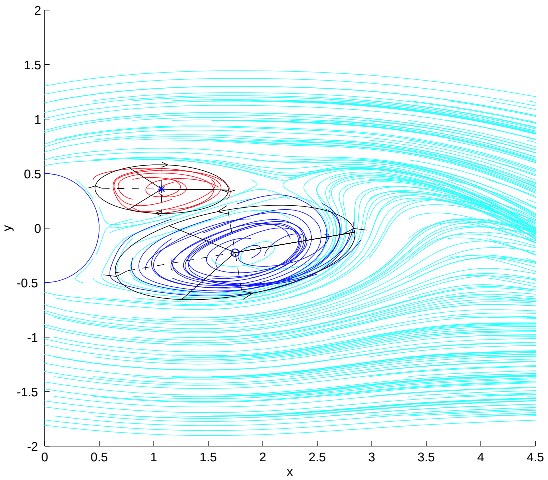

Figure 4: *Vortices detected by the winding-angle method in the wake of the tapered cylinder, [Sadarjoen & Post, 2000].*

4. Attribute-Based Feature Tracking

The vortical patterns found in the horizontal slices can be connected using a feature tracking technique. In Reinders et al.[6] we described a tracking system to track features in time, in order to determine their evolution. The tracking algorithm uses a feature data representation in order to track continuous paths.

The algorithm can be summarised as follows. Continuous paths are tracked based on a prediction-verification method. Once a path has been initialised, we predict the attributes of the feature in frame $i+1$ based on the attribute values of the features in the two preceding frames $i-1$ and i. For this purpose, we use linear behavioural rules. The attributes of the predicted feature in frame $i+1$ are tested to the attributes of the candidate features in that frame in order to find a correspondence with the prediction. Thus, the path is extended until no more correspondences can be found, or the last frame is reached.

Correspondence between two features is established by testing the feature data with certain tolerances. The feature data consists of attributes such as position, size, and mass. For each attribute set, a number of correspondence functions are evaluated, and the results of these tests are combined in a single correspondence factor. Each correspondence function returns a correspondence value C_i. The user can assign weights W_i to each function, with which the correspondence factor CF is computed as a weighted average of all correspondence values:

$$CF = \frac{\sum_i C_i * W_i}{\sum_i W_i}. \tag{1}$$

A positive correspondence factor means that a match has been found, with $CF = 1$ for an exact match. $CF < 0$ indicates no match.

A crucial observation in this study is that the same prediction- verification tracking algorithm can be applied for two purposes:

- **Space-tracking**, or finding the matching 2D vortices in adjacent horizontal slices, by finding the correspondences between the vortices found in slices i and $i+1$. This results in chains of matching 2D vortices through ranges of slices, which make up 3D vortex tubes from a series of elliptical cross sections.
- **Time-tracking** of the 3D vortex tubes through the frames of the data set, by finding the correspondences between the vortices in frames i and $i+1$. This results in chains of evolving 3D vortices through ranges of frames.

In the case of the 2D vortices detected by the winding-angle method, we used the following attributes for space-tracking (see Table 2): centre position (Eq. 2), volume (Eq. 3), and angular velocity (Eq. 4). The rotational direction can also be taken into account as a Boolean criterion, to ensure that only vortices rotating in the same direction are matched.

Attribute	Correspondence criteria	
Position $\vec{P}$	$\left\Vert \vec{P}_2 - \vec{P}_1 \right\Vert \leq T_{dist}$	(2)
Volume V	$\frac{\lvert V_2 - V_1 \rvert}{\max(V_1, V_2)} \leq T_{vol}$	(3)
Angular velocity ω	$\frac{\lvert \omega_2 - \omega_1 \rvert}{\max(\omega_1, \omega_2)} \leq T_{\omega}$	(4)

Table 2: *Correspondence criteria for the vortices.*

The correspondence functions associated with the 3D vortex tubes for time-tracking are similar to the criteria for the 2D vortices however evaluated on the following attributes: the position of the centre of gravity (eq. 2), the total volume (eq. 3), and the average rotational speed (eq. 4). Optionally, the tube topology can be taken into account.

The tracking system has been designed in such a way that it is easily extensible for tracking new types of features. The hierarchical class structure provides generic classes for attributes and attribute sets, from which new classes can be derived. By adding just three new derived classes, we were able to adapt the system for 3D vortex tracking.

5. Visualisation

We visualise the results of the tracking processes in various ways: the results of the 2D vortex extraction by the winding-angle technique are visualised using a special type of ellipse icon (see Figure 5), which is an ellipse with a number of curved spokes. This icon reflects the shape and size of a 2D vortex in the ellipse shape, the rotation speed in the number of spokes, and the rotation direction in the curvature of the spokes.

Figure 5 shows one step during the space-tracking. A number of 2D vortices at different slices are shown in the feature viewer. The figure shows one path of 2D vortices, from bottom to top, the prediction at the end of the path, and two candidates in the next slice. Clearly, one of the candidates corresponds very well to the prediction and should be added to the path.

The results of the space-tracking, consisting of sets of corresponding 2D ellipses, are visualised by assigning each set of corresponding ellipses a distinct colour, and by constructing a 3D path connecting the centres of these ellipses. This path can be conceived as an approximation of the vortex core, and gives an idea of the 3D structure of the vortex. Figure 6 shows the connecting paths representing the vortex cores. Each 3D vortex has a distinct colour.

The lower part of Figure 6 shows the event graph[7] which provides a schematic overview of the features in all frames. On the horizontal axis are the time steps, in which tracking has been performed. On the vertical axis are the features in each time step, ordered according to their number, but any feature attribute could be used. The edges in the graph indicate the correspondences that have been found between the different time steps, and therefore, a path in the graph indicates the evolution of a feature in time.

There is a direct link between the graph and the 3D view. Each node in the graph corresponds with one object in the 3D view, with the same colour. The user can select objects in the graph and view them in 3D and vice versa. Thus, for example, the user can manually select the candidate that matches the prediction in the 3D view, and verify in the graph whether the automated tracking finds the same result. Or, the user can select an unmatched (grey) feature in the graph and verify in the 3D view, why it has not been included in any path.

We visualise the results of the time-tracking, consisting of spatial paths of 3D vortices, by animating the 3D vortices, identified by their colour assigned in the space-tracking phase. This interactive, animated visualisation shows the movement and evolution of the vortices in a series of time steps (see Figure 7).

6. Results

In an earlier paper[11] we showed that the vortices in slices perpendicular to the cylinder's axis evolve consistently in time and therefore can be tracked. It is fairly easy to automatically track the 2D vortices in one slice over time with our tracking algorithm‡. The result is that, in this slice, vortices appear alternatingly on the left and right side of the cylinder, and have alternating clockwise and counterclockwise rotational directions. This is in accordance with our expectations.

For the space-tracking, we generated streamlines in 31 slices with an increasing z-coordinate value from the bottom to the top of the cylinder (with 150 streamlines per slice). For each slice, we extracted the 2D vortices with the winding-angle method and stored the attributes as a frame in our feature data hierarchy. Thus, we obtained feature data with 31 frames with 2D vortex features, where each frame corresponds to a slice at a certain z-level. This feature data can be tracked automatically by our tracking method.

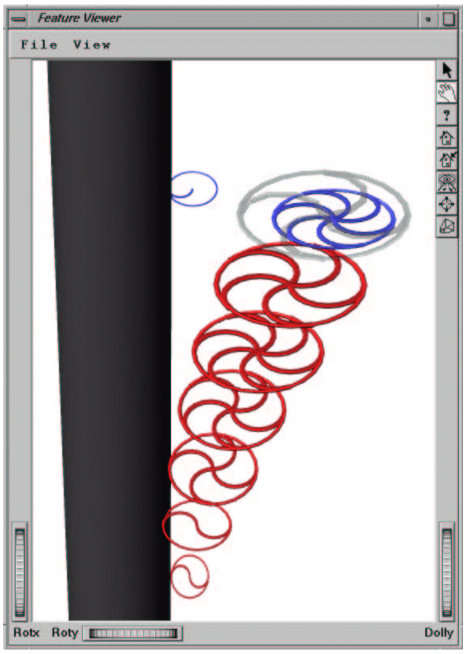

Figure 5: *Tracking 2D vortices in space (z-direction) in order to obtain 3D vortices at one instance of time.*

The tracking in the z-direction results in a number of paths, each of which represents a vortex feature in 3D. Each path in the event graph is transformed to an attribute set with a graph representation of the 3D vortices: the vortex graph. This vortex graph consists of nodes holding the 2D vortex data connected by edges. The vortex graph can be visualised by connecting the centre point positions of the 2D vortices by lines according to the graph edges, see Figure 6. This

‡ An animation with 400 frames can be found at `http://visualisation.tudelft.nl/Tracking`.

line illustrates the vortex core, while the ellipse shaped icons show the vortex geometry. Thus, the 2D vortices obtained by the winding-angle method have been transformed to 3D vortex features by tracking in space.

The resulting 3D vortices are features at one particular instance of time which can be tracked in time. We repeated the process described above for 100 frames (generating a total of $100 \times 31 \times 150 = 465.000$ streamlines) and tracked the 3D vortex graphs in time.

The result of tracking the 3D vortices in time is interesting[‡]: the vortices move from the top to the bottom, as can be seen in Figure 7. They originate at the top of the cylinder and slowly move downward until they dissipate at the bottom. This is remarkable because the streamlines all show an *upward* motion which can be expected because the tapered cylinder is smallest at the top. This upward motion can also be seen in Figure 2.

Another remarkable phenomenon in the evolution is that the vortices seem to be 'weak' somewhere in the middle of the cylinder. The vortex cores tend to show strange curves, and, perhaps as a consequence, the vortices are short (spanning fewer slices). Also this phenomenon can be clearly seen from the six frames in Figure 7.

7. Discussion and Future Work

We have applied vortex detection and tracking to the case of the flow past a tapered cylinder. Starting from an essentially 2D vortex detection technique, we have used spatial correspondence tracking to extend the 2D detection technique to 3D, which allows the 3D structure of the vortices to be derived and quantified. Next, we have applied the same tracking technique to establish temporal correspondences between these 3D vortices, which allows the motion of the vortices to be tracked. This has revealed interesting patterns.

The application shows that two essentially different visualisation techniques can be successfully combined. The tracking procedure needed only little adjustment in order to be able to track vortex features in both space and time. This shows the flexibility of the tracking system.

The success of our method in this case is based on the assumption that the vortex cores are roughly perpendicular to the slice planes, and that the projected vortex streamlines will therefore result in stable circulating patterns. In other cases, such as turbomachinery flows[9], where this assumption may not be valid, our detection method could be extended to 3D by combining it with techniques that are less sensitive to the direction of the core lines[3, 4, 9]. Also, as mentioned earlier, multiple, perpendicular slice planes could be used to find vortex cores with arbitrary orientations.

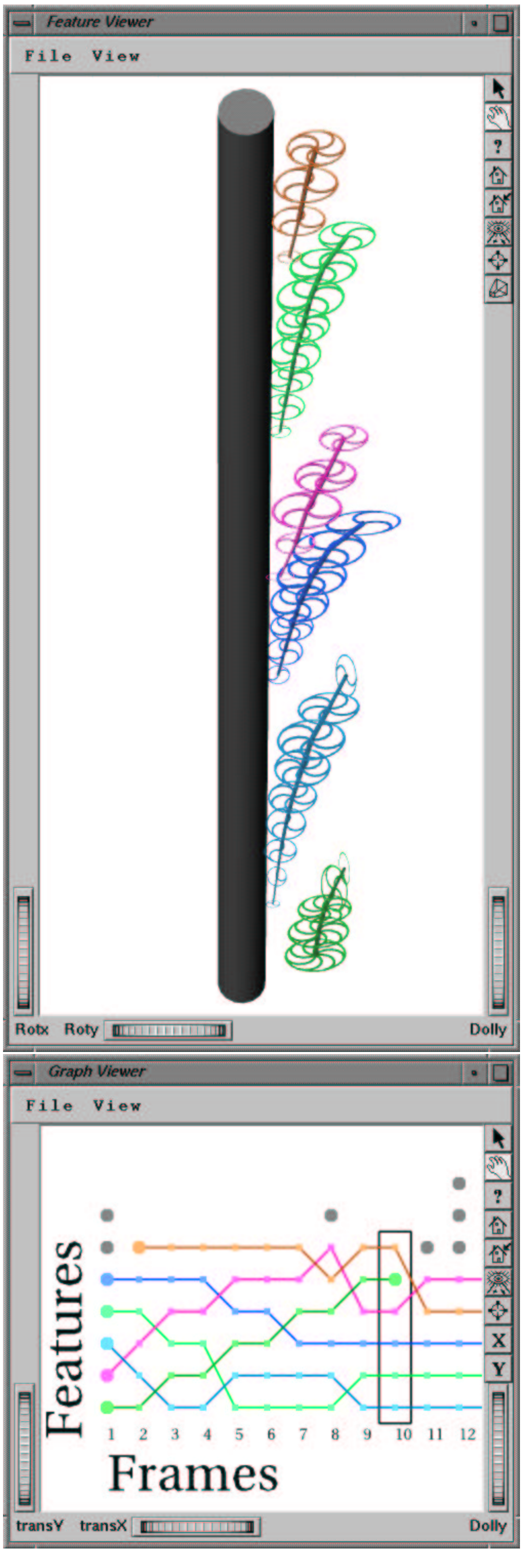

Figure 6: *The 3D vortex graphs (top) obtained by tracking 2D vortices in space. The event graph (bottom) shows correspondences of 3D vortices in time.*

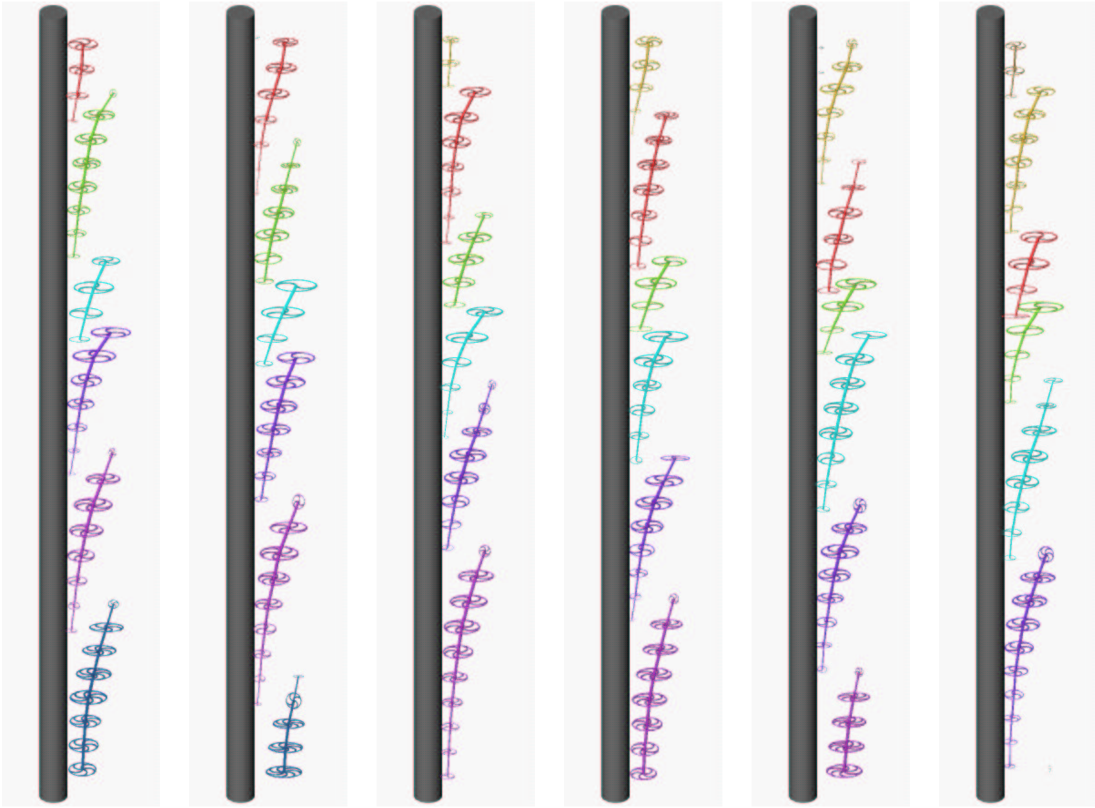

Figure 7: *Six frames from an animation sequence showing the evolution of a number of vortices. Notice how the features move from top to bottom.*

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